RESTRICTIONS ON POMERON COUPLINGS

Henry D. I. Abarbanel[†]

Weizmann Institute of Science, Dept. of Physics, Rehovoth, Israel

S. D. Ellis

National Accelerator Laboratory, Batavia, Illinois 60510

M. B. Green[*] and A. Zee[**]

The Institute for Advanced Study, Princeton, New Jersey 08540

ABSTRACT

We present restrictions on the couplings of zero mass Pomerons which follow from bounds on the absorptive parts of Reggeon particle amplitudes and the vanishing of the triple Pomeron vertex measured in inclusive processes.

When one ascribes the mechanism responsible for asymptotically constant total cross sections or diffractive behaviour in general to the presence of an isolated J-plane pole (fixed or moving) whose $t = 0$ intercept is precisely one, strong restrictions are required on the various couplings of the object associated with that pole (the Pomeron). The most ancient of these is the necessity that Pomerons of zero mass decouple in multi-Regge models for particle production in order to make them consistent with unitarity.[1] Also in the "reggeon calculus" model of high energy behaviour it appears to be necessary that certain triple Pomeron couplings vanish for the ambitious program there envisioned to be self consistent.[2] Further, it is now known that the three Pomeron vertex measured in inclusive processes must vanish when all legs carry zero mass.[3,4] In this paper, we propose to use the latter fact together with bounds on Reggeon particle absorptive parts to provide very striking constraints on Pomeron couplings.

Our starting point is to consider the W^2-channel absorptive part, $A_{a\to c}^{W}(W^2, t(0), q_a^2, q_c^2)$, of the "process": Reggeon of momentum q_a, spin $a(q_a^2)$, W^2-channel helicity a, plus a spinless particle b of mass m_b going to some system c of momentum q_c and, for simplicity, the same spinless particle b (see Figure 1). We exhibit the dependence of A^{W} on $W^2 = (q_a + p_b)^2$, $t = (q_a - q_c)^2$ evaluated at W^2- channel scattering angle 0, q_a^2, q_c^2, the W^2-channel helicity a, and some cluster variables c for the object of momentum q_c. This absorptive part has the usual representation as a sum over intermediate states

$$A^{W}_{a \to c}(W^2, t(0), q_a^2, q_c^2) = i(2\pi)^4 \sum_N T_{a+b \to N}(W^2, q_a^2) T^{*}_{c+b \to N}(W^2, q_c^2)$$

$$\times \, \delta^4(q_a + p_b - p_N), \tag{1}$$

where T is the $2 \to N$ amplitude indicated.

By using the Cauchy-Schwartz inequality on this sum, we may bound it above, within the Lehmann ellipse,[4] by the product of absorptive parts for the elastic processes $a+b \to a+b$ and $c+b \to c+b$. That is, for $0 = \psi + i\phi$ we write[5]

$$\left| A^{W}_{a \to c}(W^2, t(\psi + i\phi), q_a^2, q_c^2) \right|^2 \leq A^{W}_{a \to a}(W^2, t(i\phi), q_a^2, q_a^2) \times$$

$$\times A^{W}_{c \to c}(W^2, t(i\phi), q_c^2, q_c^2). \tag{2}$$

We want to use this in the case when the Reggeon a is a Pomeron with $q_a^2 = 0$, for then the vanishing of the leading asymptotic behaviour on the right hand side of (2) has implications for the couplings appearing on the left.

First, we recall that in an inclusive process at large incident energy, fixed missing mass and momentum transfer one measures the W^2 absorptive part of the maximum flip t-channel helicity amplitude for Reggeon particle scattering.[6] To cross from $A^{W}_{a \to a}$ for $q_a^2 = 0$ to t-channel amplitudes, we remember that the helicity crossing matrice for lightlike momenta is simple, and indeed,

$$A^{W}_{a \to a}(W^2, t, 0, 0) = A^{t}_{a, -a}(W^2, t, 0, 0), \tag{3}$$

which relation we shall want to continue to complex helicities. Now we are able to write the basic inequality (2) in terms of the t-channel amplitude measured in inclusive processes

$$\left| A^{W}_{a \to c}(W^2, t(\psi + i\phi), 0, q_c^2) \right|^2 \leq A^{t}_{a, -a}(W^2, t(i\phi), 0, 0) \times A^{W}_{c \to c}(W^2, t(i\phi), q_c^2, q_c^2),$$

$$\tag{4}$$

where $a = \pm 1$.

Let us ask for the asymptotic behaviour in W^2 of each side of (4) for the case $\phi = 0$, so $t = 0$ on the right hand side. Further, the Reggeon is taken to be the Pomeron which we assume to be a pole with $a_p(0) = 1$. We know from the arguments in Ref. 3 that the asymptotic behaviour of $A^t_{1,-1}$ is not $(W^2)^{a_p(0)}$ since the three Pomeron coupling in that helicity configuration vanishes for $q^2_a = 0$, $t = 0$, so we must have $A^t_{1,-1} \sim (W^2)^{a_N(0)}$ where $a_N(0)$ is the next singularity we encounter and would most likely be $\approx \frac{1}{2}$. Now the asymptotic inequality reads

$$g^2_{PcP}(0, q^2_c, t)(W)^{2a_P(t)} \leq g_{PPN}(0,0,0)\, g_{ccP}(q^2_c, q^2_c, 0) \times (W^2)^{a_P(0) + a_N(0)},$$
(5)

where the couplings g (with W^2-channel helicity indices) incorporate the Reggeon/two particle vertices which we assume not to vanish. In the regime where the inequality (4) holds, which is essentially all space-like t, whenever

$$2a_P(t) > a_P(0) + a_N(0),$$
(6)

(5) requires that

$$g_{PcP}(0, q^2_c, t) = 0$$
(7)

In other words, a Pomeron of zero mass and helicity ± 1 must decouple from any state of mass q^2_c and another Pomeron subject to (6)[7] (Figure 2). This is, in a sense, a model independent generalization of the results of Ref. 1.

The coupling in (5) or (7) refers all helicities to the W^2-channel, but since the transition to any helicity state of c must vanish, we can conclude immediately that (7) holds for t-channel helicity vertices as

well (bearing Eq. (3) in mind). Furthermore, if the system c is characterized by a definite spin (which may be a function of q_c^2) and helicity, then we may use general Lorentz properties of three particle or Reggeon couplings to conclude from Eq. (7) that the coupling $g_{PPc}(0, t, q_c^2)$ (which appears in the Pomeron-particle scattering amplitude $P(0) + b \to P(t) + b'$ where c is exchanged and $(p_b - p_{b'})^2 = q_c^2$) also vanishes. These remarks will be elaborated on in an expanded version of this paper.

On the basis of this last observation, we may go back to the Pomeron particle absorptive part $A_{1\to1}^W(W^2, 0, 0, 0)$ and note that no t-channel Reggeon or particle exchange can contribute to its asymptotic behaviour. Hence, the absorptive part $A_{1\to1}^W(W^2, 0, 0, 0)$ decreases <u>faster than any power</u> of W^2 as $W^2 \to \infty$. Thus, we may write

$$\int_{m_b^2}^\infty dW^2 (W^2 - m_b^2) \, \tilde{A}_{1\to1}^W(W^2, 0, 0, 0) = R \qquad (8)$$

where $\tilde{A}_{1\to1}^W$ is $A_{1\to1}^W$ with the W^2-channel kinematical singularities removed and R denotes the possible contribution of a nonsense right signature fixed pole[8] at $J = 0$. In the absence of such a fixed pole (R = 0) we have, since $\tilde{A}_{1\to1}^W(W^2, 0, 0, 0)$ is positive semidefinite,[9]

$$(W^2 - m_b^2) \, \tilde{A}_{1\to1}^W(W^2, 0, 0, 0) = 0 \qquad (9)$$

In other words, a zero mass, helicity ± 1 Pomeron couples a particle only to itself and not to any other state with different mass.

In the inclusive process $a + b \to c +$ anything at incident energy s, momentum transfer q^2, and missing mass W, the asymptotic behavior arising from Pomeron exchange (in the a-c channel) is

$$s^2 \frac{d\sigma}{dq^2 dW^2} (a+b \to c+\text{anything}) \underset{\substack{q^2, W^2 \\ \text{fixed}}}{\overset{s \to \infty}{\sim}} \frac{|\beta_{acP}(q^2)|^2}{16\pi} s^{2a_p(q^2)} F(q^2, W^2) \quad (10)$$

where $\beta_{acP}(q^2)$ is the two particle-Pomeron coupling and $F(q^2, W^2)$ is proportional to $\tilde{A}^t_{a(q^2), -a(q^2)}(W^2, 0, q^2, q^2)$. In the absence of a $J = 0$ fixed pole $F(q^2, W^2)$ will vanish at $q^2 = 0$ unless $W^2 = m_b^2$. In the presence of a $J = 0$ fixed pole, $F(q^2 = 0, W^2)$ decreases faster than any power as $W^2 \to \infty$ and thus in any case the diffractive production of large missing mass will be suppressed at $t = 0$. There is some evidence[10] that diffractive production of the $N^*(1512)$ and $N^*(1688)$ resonances in πp collisions "dips" at $q^2 = 0$. The data are hardly compelling on this point, though they are suggestive. The evidence at this time is against such a forward dip in diffractive production of the Roper $N^*(1405)$.

There are some other conclusions which one may draw on the basis of our arguments. First, since the right hand side of (4) at $\phi = 0$ must decrease faster than any power of W^2, so must the left. If d is an object which may be exchanged in the process Pomeron (q_a, helicity $= \pm 1$) + b $\to$ c+b, then the coupling $g_{Pcd}(0, q_c^2, q_d^2)$ vanishes[11] for $q_d^2 \leq 0$.

Helicity conservation in the Pomeron-Reggeon-Reggeon vertex requires the net helicity change, (c-d), to equal ± 1. Of experimental interest is the vertex measured in inclusive reactions where $q_d^2 = q_c^2 \equiv q^2 \leq 0$ and where c and d have helicities, $c = a_R(q^2) = -d$. Thus, when $a_R(q^2) = \frac{1}{2}$ our result requires the vanishing of $g_{Pa_R(q^2)a_R(q^2)}(0, q^2, q^2)$ which is the coefficient of the $(s/W^2)^{2a_R(q^2)} (W^2)^{a_P(0)}$ term in Eq. 10 in the large s/W^2, and large W^2 limit. Phenomenologically,[12] this particular term

(corresponding to $g_{f f P}$) appears to be quite sizeable for non-zero t.

Our discussion suggests an analogy between the Pomeron couplings and those of the photon. In electroproduction one measures $W_2(q^2, W^2)$ in the limit prescribed in (10). From the fact that the photon is coupled to a conserved current, it follows that $W_2(0, W^2)$ vanishes unless $W^2 = m_p^2$. One may entertain the speculation that in some sense a zero mass Pomeron behaves as if it couples to a conserved current. A host of questions, such as whether or not there is a conserved charge associated with such a current, are opened up. It goes beyond the bounds of this note to even begin to properly answer these.

ACKNOWLEDGEMENT

REFERENCES

1. J. Finkelstein and K. Kajantie, Phys. Letters 26B, 305 (1968) and Sov. Phys. JETP 19, 1148 (1964).

2. V. N. Gribov and A. A. Migdal, Yad. Fiz. 8, 1002 (1968) [Sov. J. Nucl. Phys. 8, 583 (1969)].

3. H. D. I. Abarbanel, G. F. Chew, M. L. Goldberger, and L. M. Saunders, Phys. Rev. Letters 26, 937 (1971). See also the proof in C. E. de Tar and J. H. Weis, Phys. Rev. D4, 3141 (1971).

4. G. Mahoux and A. Martin, Phys. Rev. **174**, 2140 (1968).

5. Some readers will be as concerned as we are that the demonstration of these bounds requires one to treat the Reggeon no differently from an ordinary particle and write a c o n v e n t i o n a l unitarity relation involving a sum over states. When we were writing this paper, we received a preprint from A. R. White, Univ. of Cambridge, D. A. M. T. P. 72/19, which provides a rather rigorous justification for this from extended unitary. See also H. D. I. Abarbanel, NAL preprint THY-28 for a discussion of Reggeon absorptive parts.

6. de Tar and Weis, Ref. 3.

7. This result is exhibited by the Dual Resonance model with unit intercept leading trajectory (the Ghost-free model).

8. The presence of such nonsense right signature fixed poles in strong amplitudes is normally forbidden by unitarity in the t-channel. However, in the approximation of neglecting Regge cuts (which may be necessary to define Reggeon couplings) t-channel unitarity may assume a linear form as in photon processes, in which case a fixed pole may appear. There is some phenomenological evidence for the absence of such a fixed pole at finite q^2, S. Ellis and A. Sanda, NAL preprint THY-49.

9. This relation has been discussed in a somewhat different context by M. B. Einhorn, M. B. Green, and M. A. Virasoro "Duality in Diffraction Dissociation" L. B. L. preprint.

10. E. W. Anderson et al., Phys. Rev. Letters **25**, 699 (1970).

11. This is similar to a result reported in private conversations by F. E. Low and G. Veneziano with some of the authors.

12. For example, J. M. Wang and L. L. Wang, Phys. Rev. Letters $\underline{26}$, 1287 (1971); P. D. Ting and H. J. Yesian, Physics Letters $\underline{35B}$, 321 (1971); S. D. Ellis and A. Sanda, NAL preprint THY-30.

FIGURE CAPTIONS

Figure 1: The absorptive amplitude for a Reggeon of momentum q_a, spin $\alpha(q_a^2)$, W^2-channel helicity a, plus a spinless particle b to go to some system of things of momentum q_c and b. This is a function of the momentum transfer $t = (q_a - q_c)^2$ and the energy variable $W^2 = (q_a + p_b)^2$.

Figure 2: The coupling of a zero mass, helicity ± 1 Pomeron to another Pomeron of $(mass)^2 = t$ and the system of momentum q_c. The vanishing of the triple Pomeron vertex measured in inclusive reactions requires this vertex to be zero.
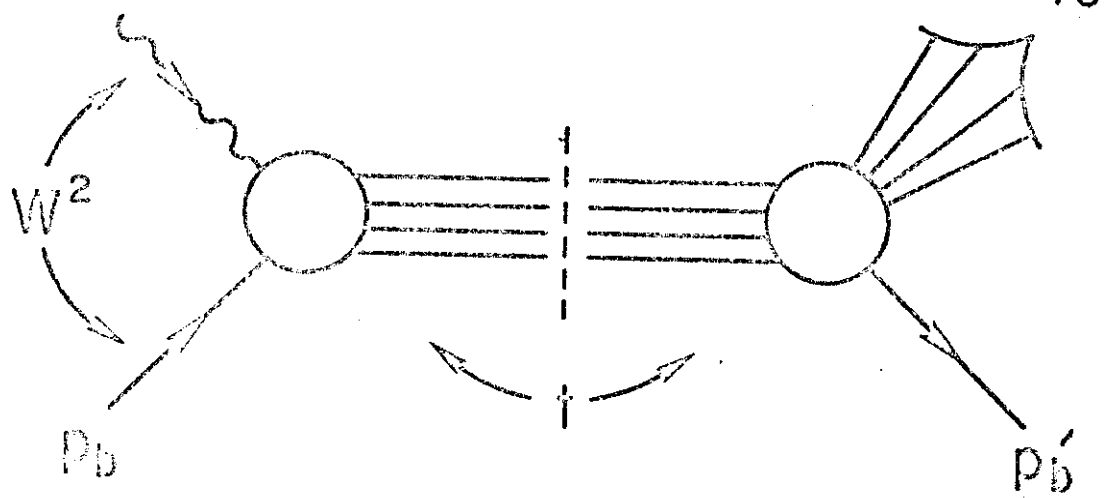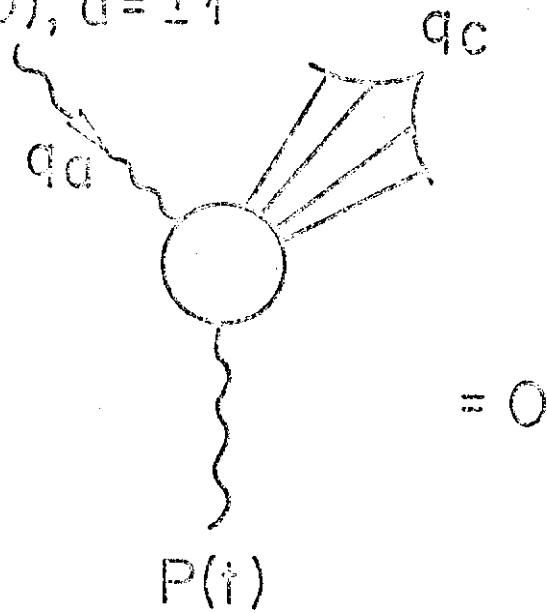
$q_a, \alpha(q_a^2), a$

q_c

W^2

P_b

P_b'

Fig. 1

$P(q_a^2 = 0), a = \pm 1$

q_c

q_a

$= 0$

$P(\dagger)$

Fig. 2